**Discrete Math Homework 13**

**Due Wednesday, April 19 at the beginning of class**

General instructions:

* Use standard size paper (8.5 by 11).
* Answer each question in order using a single column.
* Be neat. If we cannot read your solution it is wrong.
* Show your work. If you just write an answer, you will get minimal credit even if the answer is correct.

**A little more counting**

**Question A)** How many 5-card hands (combinations) are there that are four of a kind. (Four cards of the same rank, and a fifth card that is a different rank.)

Task 1: Select 2 ranks C(13, 2)=13x6

Task 2: Select rank of quad C(2, 1)=2

Task 3: Select suits of quad C(4,4) = 1

Task 4: Select suit of single C(4,1) = 4

Product Rule: 13x6x2x1x4 = 624

**Question B)** How many 5-card hands (combinations) are there that are two pair. (Two card of one rank, two cards of a second rank, and a fifth card that is a third rank.).

Task 1: Select 3 ranks C(13, 3)=13x2x11

Task 2: Select ranks of doubles C(3, 2)=3

Task 3: Select suits of one double C(4,2) = 6

Task 4: Select suits of one double C(4,2) = 6

Task 5: Select suit of single C(4,1) = 4

Product Rule: 13x2x11x3x6x6x4 = 123,552

Two pair is much more common than four of a kind.

**Question C)** Fill in the following table to decide the probability that when you roll two dice the sum is a prime number. We are pretending that the dice are distinguishable and form a sequence of rolls. Check each square if the sum is prime. The probability will be the number of squares with a check divided by the total number of squares.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 |
| 1 | x | x |  | x |  | x |
| 2 | x |  | x |  | x |  |
| 3 |  | x |  | x |  |  |
| 4 | x |  | x |  |  |  |
| 5 |  | x |  |  |  | x |
| 6 | x |  |  |  | x |  |

**Note that the sums are the same on the / diagonals.**

**Probability = 15/36 is approximately 42%**

**Rosen section 8.2**

**Question D)** Rosen 8.2 Exercise 3b, c (p. 524)

**b)** an =an−1 for n≥1, a0 =2

Assume and plug into the recurrence



Divide to get characteristic equation



Create a linear combination of the r's found



Use initial conditions to solve for constants



or



Write the final solution



**c)** an =5an−1 −6an−2 for n≥2, a0 =1,a1 =0

Assume and plug into the recurrence



Divide to get characteristic equation



Solve for r. Can use quadratic formula or factor the LHS to find that

r = -2, -3

Create a linear combination of the r's found



Use initial conditions to solve for constants





Simplifying





Solve first equation for alpha



Substitute into the second equation and solve for beta







Back solve for alpha



Write the solution



**Rosen section 8.3**

**Question E)** Instead of doing a binary search on a sorted list, we can do a trinary search instead. Here is the algorithm:

***T-Search*** *(List, target)*

*Let n = length of the list*

*Let mid1 = n/3*

*Let mid2 = 2n/3*

*If the size of the list is 1*

*Return true if we found the target or false if not.*

*If target < List[mid1] return* ***T-Search****(First third of List, target)*

*Else if target < List[mid2] return* ***T-Search****(Second third of List, target)*

*Else return* ***T-Search****(Last third of List, target)*

a) Argue why the recurrence relation for the work W(n) performed by this function is W(n) = W(n/3) + 2 with W(1) = 1

When we do the function we divide the list into thirds. We do at most two comparisons (+2) to decide which third of the list to call recursively W(n/3). If our list has only one value in it, we can decide if the value is in the list with just one comparison. W(1) = 1.

b) Use the Master theorem from page 582 to find the Big-O for W(n)

We need to identify the values a, b, c and d.

a=1 b=3 c=2 d=0

We compare a with  gives

1 vs 

We are in the second case and we get



**Question F)** Consider the recursive Merge sort

***Merge-Sort*** *(List)*

*Let n = length of the list*

*If n is < 2 return List*

*Else*

*Let L1 = first half of the list*

*Let L2 = second half of the list*

*Let L1' =* ***Merge-Sort****(L1)*

*Let L2' =* ***Merge-Sort****(L2)*

*Let L' = Merge(L1', L2')*

*Return L'*

The recurrence relation for Merge sort is M(n) = 2M(n/2) + cn

a) Use the Master theorem from page 582 to find the Big-O for M(n)

We need to identify the values a, b, c and d.

a=2 b=2 c=c d=1

We compare a with  gives

2 vs 

We are in the second case and we get



b) One tricky bit, is that the merge operation uses extra memory to copy the values from L1' and L2' into the result list L'. If we want to do the merge without using any extra memory, the time for the merge is no longer proportional to n. A naïve in-place merge would be n-squared giving a recurrence relation of



Use the Master theorem from page 582 to find the Big-O for Q(n).

We need to identify the values a, b, c and d.

a=2 b=2 c=c d=2

We compare a with  gives

2 vs 

We are in the first case and we get



c) Suppose that we could do the in-place merge with time that is proportional to , that would result in a recurrence relation of



Use the Master theorem from page 582 to find the Big-O for R(n).

We need to identify the values a, b, c and d.

a=2 b=2 c=c d=5/4

We compare a with  gives

2 vs 

We are in the first case and we get



Note that this is not as good as nlogn.

**You may choose to solve one (and only one) of the following Extra Credit Problems. If you submit more than one, only the first will be graded.**

**Extra Credit 1)** Compute the probability of being dealt a full house, four of a kind and two pair. (Divide by the number of combinations that could be dealt to you, which is C(52, 5).

C(52,5) = 52x51x50x49x48/5x4x3x2x1 = 52x17x10x49x6=2,598,960

P(full house) = 3,744/2,598,960 = 0.0014405762304922 (one in 694 hands)

P(four of a kind) = 624/2,598,960 = 0.000240096038415 ( one in 4,165 hands)

P(two pair) = 3,744/2,598,960 = 0.047539015606242 ( one in 21 hands)

**Extra Credit 2)** Rosen 8.2 Exercise 45 (p. 526)

Suppose that each pair of a genetically engineered species of rabbits left on an island produces two new pairs of rabbits at the age of 1 month and six new pairs of rabbits at the age of 2 months and every month afterward. None of the rabbits ever die or leave the island.

**a)** Find a recurrence relation for the number of pairs of rabbits on the island n months after one newborn pair is left on the island.

**b)** By solving the recurrence relation in (a) determine the number of pairs of rabbits on the island n months after one pair is left on the island.